APPLICATION OF WAVELET BASED DISCRETIZATION METHOD TO ELASTICITY AND PLASTICITY PROBLEMS

J. Majak¹, M. Pohlak², and M. Eerme³

Department of Mechanics
Tallinn University of Technology
Ehitajate tee 5, 19086 Tallinn, Estonia
¹E-mail: jmajak@staff.ttu.ee
²E-mail: meelisp@staff.ttu.ee
³E-mail: eerme@staff.ttu.ee

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Summary. Haar wavelet based discretization method for determination of the stress-strain state of the solid structures is developed. A linear-elastic and rigid-plastic material models are considered. Both, an analysis and optimal design problems are included.

1 INTRODUCTION

A wavelet is a basis function used to construct a wavelet transform. The Haar wavelet is the first known wavelet and was proposed in 1909 by Alfred Haar. Haar wavelet theory has been applied to various problems including signal processing in communications, image compression-extraction, solution of the linear and nonlinear integral equations etc. Wavelet discretization technique is adopted for solving some solid mechanics problems in [1-5]. However, unfoundedly little attention has been paid to elasticity and plasticity problems. An operational matrix of integration is introduced in [6,7] for Haar and Legendre wavelets, respectively. Haar analysis of the dynamic systems got impulse in 1997, when Hsiao [8] developed the Haar product matrix and coefficient matrix. These results form a good presumption for treatment an effective discretization methods.

2 AIM OF THE STUDY

Aim of the current study is to develop and improve wavelet based discretization method for solving the elasticity and plasticity problems. Results obtained at the moment are related mainly to analysis of one- and two dimensional linear elasticity problems, some simple variational problems. However, the list of new features needed to include is remarkable: material anisotropy, geometrical nonlinearity, physical nonlinearity, variational problems and etc.

Especially attention is paid to variational problems in the following reasons:
- some few papers available, covering application of wavelet based discretization methods to variational problems, have successful results,
- similarly to FEM approach, the elasticity and plasticity problems can be formulated as minimization (maximization) of the functional.

3 MAIN RESULTS OF THE STUDY

Haar wavelet based discretization method for determination of the stress-strain state of the solid structures is developed. A linear-elastic and rigid-plastic material models are considered. Both, an analysis and optimal design problems are included. The governing equations and boundary conditions are first normalized and discretized by employing Haar wavelet basis. In literature the Haar wavelet based discretization technique is applied directly to governing differential equations. In the current paper the both, strong and weak formulations of the differential equations are considered. The CAS-es (Computer Algebra Systems) method is utilized for solving posed problems. In the case of nonlinear equations of motion the obtained algebraic system of equations is solved by Newton method. An algorithm is implemented in MAPLE 10 code. Composed symbolic-numerical algorithm
allows to derive algebraic system of equations using symbolic calculation and solve it numerically. The partial
derivatives, needed for application of the Newton method are derived explicitly using symbolic calculation.
Obtained results are compared with Runge-Kutta type methods, FEM, FDM and others, also with exact solutions
(if exists). The current results and the results obtained by alternate methods are found to be in a good agreement.

Some conclusions of the work done can be formulated as:

- the Haar wavelet discretization technique is applied to weak form of differential equations and the
  weight function is determined in accordance with Galjorkin method,
- the solution corresponding to strong formulation of differential equations is given, the results are
  compared and analyzed, the advantages of the weak formulation are obvious,
- discretization method developed appears fast and especially effective for solving boundary value
  problems,
- unique algorithms for calculating matrices H and P are present,
- Haar transform is relatively simple because the set of Haar functions forms a local basis - each function
  contains just one wavelet and remains to be zero elsewhere in interval,
- employing CAS-es method makes an algorithm more flexible to changes in problem formulation

Haar wavelet based discretization method developed is improved on various solid mechanics problems.

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