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ON THE MICROMECHANICAL RESPONSE OF FUNCTIONALLY GRADED STRATIFIED MEDIA

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Summary. *The aim of this contribution is to formulate and apply a mathematical model of a certain class of functionally graded composites. The proposed model makes it possible to investigate the effect of microstructure size on the macroscopic composite behavior.*

1 INTRODUCTION

The subject of this contribution is a macroscopic description and analysis of the elastodynamic response of certain two-phase functionally graded stratified materials (FGM). The medium under consideration is made of a large number of thin layers, Fig. 1.

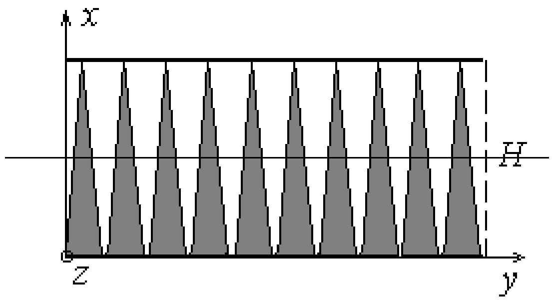

Figure 1. Scheme of FGM.

Every layer has a constant thickness λ, and is composed of two homogeneous sublayers with slowly varying thicknesses $\lambda_a(x)$, a =1,2. Hence, $\lambda = \lambda_1(x) + \lambda_2(x)$ for every $x \in$ [0,H] where H is a thickness of the FGM under consideration. It means that the material structure is periodic in a direction normal to the x-axis with a constant period λ but volume fractions $\lambda_a(x),/\lambda$, a =1,2, $x \in$ [0,H] are slowly varying in the direction of the x-axis. It is assumed that every homogeneous sublayer is linear elastic and every two adjacent sublayers are perfectly bonded. It means that on the microstructural level the elastic response of FGM under consideration is described by PDEs with non-uniformly oscillating functional coefficients.

2 ANALYSIS

The main aim of this contribution is to formulate a mathematical model, which describes the elastic response of the above FGM. In contrast to the models obtained by homogenization of the aforementioned PDEs, [1], cf. Chapter 6, the proposed modeling technique leads to the equations involving the period length as a certain microstructural parameter. Hence, the derived model equations are able to investigate some micromechanical phenomena. We can mention here boundary layer fluctuations in strains and stresses. Moreover, coefficients in the resulting macroscopic model equations are slowly varying functions of argument $x \in [0,H]$. The proposed modeling technique is a certain generalization of the tolerance averaging approach which for microperiodic materials and structures was detailed in [2] and applied in a series of papers (cf. [2] and the list of references therein). The tolerance averaging technique takes into account some approximations in computations of macroscopic characteristics of FGM. Independently of this technique the modeling is based on the heuristic assumption that the distribution of displacement field w on the microstructural level can be decomposed into the form $w = u + gv$ where g is the known continuous piecewise linear periodic function. Fields u, v are basic kinematic unknowns being slowly varying across the thickness of every layer. Field u represents the macroscopic (averaged) part of total displacement w and field v constitutes the fluctuation amplitudes of w caused by the microheterogeneity of the material structure. In contrast to the periodic microstructure of a laminated medium, function g also depends on argument $x \in [0,H]$ being for every value of x a certain saw-like function well known in the modeling of laminates. The model equations for u and v have slowly varying functional coefficients depending only on argument x. The main feature of the model is that some of PDE's coefficients depend on the period λ. Neglecting these terms we obtain the locally homogenized model of the medium under consideration.

3 APPLICATIONS

The proposed model equations will be applied to the analysis of elastic response on both the macro- and microlevel in special problems. The attention is focused on the description of some microstructured phenomena. To this end a decoupling of the elastic response on the macro- and microlevels will be carried out. In this way a certain boundary layer type equation is derived. This equation makes it possible to analyze some near-boundary and near-initial phenomena related to the microfluctuations in boundary and initial conditions. An illustrative example of the general modeling results will be discussed.

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