

## OPTIMAL STRUCTURAL DESIGN WITH COMPOSITES: FREE MATERIAL AND LAMINATE DESIGN

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**Summary.** *This presentation deals with two types of approaches to the design of the topology of composite structures. One is the free-material approach where design is performed over all possible elasticity tensors and where manufacturing considerations are left for a post-processing procedure. The other is to consider directly the integer valued problem of ply lay-up of laminates with discrete values of the ply angles. Modelling and computational aspects of both approaches will be surveyed and recent results show the results that can be obtained.*

### 1 OVERVIEW

Composites are today an integral part of structural design and the next generation of commercial airlines will see the first complete fuselages made from composites. Design optimization is a natural tool for structural design using composites and over the last decade a substantial amount of work has been performed in developing tools for lay-up design of laminates used for plate and shell structures [7]. The problems are perhaps most naturally dealt with using an integer valued modelling of the design space, meaning that for example genetic algorithms are often applied (see, for example, [1] and references therein). Relaxation schemes (in the sense of mathematical programming) using ideas from topology design of continuum structures have been proposed recently and have successfully been used for design a wind turbine blades (wings), cf., [12], [14], [15]. Thus continuous optimization can be used and integer valued designs are obtained through penalization. For design problems with stiffness and free vibration requirements a parameterization using the so-called lamination parameters is in some cases useful as convex optimization problems are obtained. This is also a relaxation scheme but this is now related to overall effective stiffnesses, constrained so as to remain feasible as physical realizable laminates (in some rather general sense). After finding an optimal design, concrete lay-ups have to found by post-processing (see, for example, [11] and references therein).

The concept of lamination parameters is to extend the design space to the widest set of possible stiffness tensors that can be realized by a laminate lay-up<sup>1</sup>. Taking this idea one step further is to consider the full set of thermo-elastically admissible rigidity tensors which amounts to all positive definite and symmetric stiffness tensors (matrices) ([5]); this again results in convex problems for single and multiple load minimum compliance problems ([4]) and means that effective algorithms can be devised that can handle large scale problems ([16], [2]). The method predicts both placement of material ([13], [6]), as well as the optimal use of microstructure and is thus useful for generating tape lay-up for composite structures ([8]). That topology is also predicted was used for parts of the new Airbus 380.

<sup>1</sup>Note that the characterization of the set of lamination parameters for a fully coupled membrane-bending case is not straightforward.

## 2 PRESENTATION

The presentation will start by a brief overview of the mathematical structure of design problems formulated in terms of parameterizations of the rigidity tensor. For the free material design the main emphasis is on how this structure can be exploited in order to generate equivalent formulations that are more useful for optimization. Semi-definite programming formulations ([10]) play here a significant role and formulations that are generalizations of certain problem statements for truss design ([3]) have been useful when exploiting modern mathematical programming software ([9]). To contrast this method, a deterministic mathematical programming approach to lay-up design will also be considered. Here branch-and-bound is applied. The unifying aspect of both approaches is that the basis for the developments is a simultaneous design and analysis formulation of the optimization problem. This means that analysis is handled by the optimization algorithm and design sensitivity analysis in its classical sense is part of the computational procedure.

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